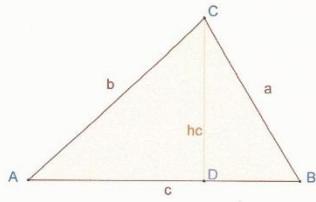


Trigonometrie – Berechnungen in schiefwinkeligen Dreiecken

Lösungsblatt 1

Berechnen Sie in folgenden Beispielen die gesuchten Größen!

△ ABC: $a = 42 \text{ mm}$, $c = 60 \text{ mm}$, $\alpha = 43^\circ$; gesucht: γ , β , b , h_c und A !



Sinussatz für die Berechnung der Winkel und der Seite b!

h_c aus dem ▲ ADC

$$\begin{aligned}\frac{a}{\sin \alpha} &= \frac{c}{\sin \gamma} \\ a \cdot \sin \gamma &= c \cdot \sin \alpha \\ \sin \gamma &= \frac{c \cdot \sin \alpha}{a} \\ \sin \gamma &= \frac{60 \cdot \sin 43^\circ}{42} \\ \sin \gamma &= \frac{60 \cdot 0,68...}{42} \\ \sin \gamma &= 0,97... \\ \underline{\gamma = 76,97^\circ}\end{aligned}$$

$$\begin{aligned}\beta &= 180^\circ - 43^\circ - 76,97^\circ \\ \underline{\beta = 60,02^\circ}\end{aligned}$$

$$\begin{aligned}\frac{a}{\sin \alpha} &= \frac{b}{\sin \beta} \\ b &= \frac{a \cdot \sin \beta}{\sin \alpha} \\ b &= \frac{42 \cdot \sin 60,02^\circ}{\sin 43^\circ} \\ b &= \frac{42 \cdot 0,86...}{0,68...} \\ \underline{b = 53,34 \text{ mm}}\end{aligned}$$

$$\begin{aligned}A &= \frac{1}{2} \cdot a \cdot c \cdot \sin \beta \\ A &= \frac{1}{2} \cdot 42 \cdot 60 \cdot \sin 60,02^\circ \\ A &= \frac{1}{2} \cdot 42 \cdot 60 \cdot 0,86... \\ A &= 1091,41 \text{ mm}^2 \\ \underline{A = 10,9141 \text{ cm}^2}\end{aligned}$$

$$\begin{aligned}h_c &= b \cdot \sin \alpha \\ h_c &= 53,34 \cdot \sin 43^\circ \\ h_c &= 53,34 \cdot 0,68... \\ \underline{h_c = 36,37 \text{ mm}}\end{aligned}$$

△ ABC: $b = 77 \text{ m}$, $\beta = 65^\circ$, $\gamma = 42^\circ$; gesucht: a , a , c und A !

$$\begin{aligned}\alpha &= 180^\circ - 65^\circ - 42^\circ \\ \underline{\alpha = 73^\circ}\end{aligned}$$

$$\begin{aligned}\frac{a}{\sin \alpha} &= \frac{b}{\sin \beta} \\ a &= \frac{b \cdot \sin \alpha}{\sin \beta} \\ a &= \frac{77 \cdot \sin 73^\circ}{\sin 65^\circ} \\ a &= \frac{77 \cdot 0,95...}{0,90...} \\ \underline{a = 81,24 \text{ m}}\end{aligned}$$

$$\begin{aligned}\frac{b}{\sin \beta} &= \frac{c}{\sin \gamma} \\ c &= \frac{b \cdot \sin \gamma}{\sin \beta} \\ c &= \frac{77 \cdot \sin 42^\circ}{\sin 65^\circ} \\ c &= \frac{77 \cdot 0,66...}{0,90...} \\ \underline{c = 56,84 \text{ m}}\end{aligned}$$

$$\begin{aligned}A &= \frac{1}{2} \cdot b \cdot c \cdot \sin \alpha \\ A &= \frac{1}{2} \cdot 77 \cdot 56,84 \cdot \sin 73^\circ \\ A &= \frac{1}{2} \cdot 77 \cdot 56,84 \cdot 0,95... \\ A &= 4185,43 \text{ m}^2 \\ \underline{A = 41 a 85,43 \text{ m}^2}\end{aligned}$$

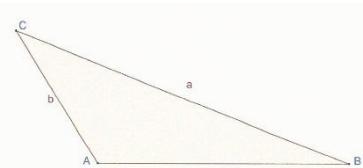
△ ABC: $a = 105 \text{ m}$, $b = 47 \text{ m}$, $\alpha = 117^\circ$; gesucht: β , γ , c und A !

$$\begin{aligned}\frac{a}{\sin \alpha} &= \frac{b}{\sin \beta} \\ \sin \beta &= \frac{b \cdot \sin \alpha}{a} \\ \sin \beta &= \frac{47 \cdot \sin 117^\circ}{105} \\ \sin \beta &= \frac{47 \cdot 0,89...}{105} \\ \sin \beta &= 0,39... \\ \underline{\beta = 23,5^\circ}\end{aligned}$$

$$\begin{aligned}\gamma &= 180^\circ - 117^\circ - 23,5^\circ \\ \underline{\gamma = 39,49^\circ}\end{aligned}$$

$$\begin{aligned}\frac{a}{\sin \alpha} &= \frac{c}{\sin \gamma} \\ c &= \frac{a \cdot \sin \gamma}{\sin \alpha} \\ c &= \frac{105 \cdot \sin 39,49^\circ}{\sin 117^\circ} \\ c &= \frac{105 \cdot 0,63...}{0,89...} \\ \underline{c = 74,94 \text{ m}}\end{aligned}$$

$$\begin{aligned}A &= \frac{1}{2} \cdot a \cdot b \cdot \sin \gamma \\ A &= \frac{1}{2} \cdot 105 \cdot 47 \cdot \sin 39,49^\circ \\ A &= \frac{1}{2} \cdot 105 \cdot 47 \cdot 0,63... \\ A &= 1569,19 \text{ m}^2 \\ \underline{A = 15 a 69,19 \text{ m}^2}\end{aligned}$$



Sinussatz!