Arithmetik – Algebraische Gleichungen höheren Grades

Lösen Sie folgende Gleichungen über die Grundmenge G = C!

*) Substitutionsmethode!

$(x^2 - 4x)^2 - 3x.(4 - x) = 4$
$(x^2 - 4x)^2 + 3(x^2 - 4x) = 4$ → → → *1. Schritt:* *Man setzt für $(x^2 - 4x) \rightarrow u$ $(x^2 - 4x) = u$*

→ → → *2. Schritt:* *Lösung der quadratischen Gleichung!*

$u^2 + 3 . u = 4$
$u^2 + 3u - 4 = 0$

$$u_{1,2} = \frac{-3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 + 4}$$

$$u_{1,2} = \frac{-3}{2} \pm \sqrt{\frac{9}{4} + \frac{16}{4}}$$

$$u_{1,2} = \frac{-3}{2} \pm \frac{5}{2}; \quad \underline{u_1 = +1;} \quad \underline{u_2 = -4;}$$

→ → →*3. Schritt:* *Die Werte für u_1 und u_2 in die Gleichung $(x^2 - 4x)$ einsetzen!*

$(x^2 - 4x) = +1 \rightarrow x^2 - 4x - 1 = 0$

$$x_{1,2} = \frac{4}{2} \pm \sqrt{\left(\frac{4}{2}\right)^2 + 1}; \quad \mathbf{x_{1,2} = +2 \pm \sqrt{5};}$$

$\underline{\mathbf{x_1 = -0{,}24;}}$ $\underline{\mathbf{x_2 = +4{,}76;}}$

$(x^2 - 4x) = -4 \rightarrow x^2 - 4x + 4 = 0$

$$x_{3,4} = \frac{4}{2} \pm \sqrt{\left(\frac{4}{2}\right)^2 - 4}; \quad \underline{\mathbf{x_{3,4} = +2}}$$

$\underline{\mathbf{L = \{-0{,}24, +2, +4{,}76\}}}$

$x^4 - 13x^2 + 36 = 0 \rightarrow x^2 = u$

$u^2 - 13u + 36 = 0$

$$u_{1,2} = = \frac{13}{2} \pm \sqrt{\left(\frac{13}{2}\right)^2 - 36}$$

$$u_{1,2} = \frac{13}{2} \pm \sqrt{\frac{169}{4} - \frac{144}{4}}$$

$$u_{1,2} = \frac{13}{2} \pm \sqrt{\frac{25}{4}}; \quad u_{1,2} = \frac{13}{2} \pm \frac{5}{2}$$

$\mathbf{u_1 = +9;}$ $\mathbf{u_2 = +4;}$ $x^2 = u;$

$x^2 = +9$ $x^2 = +4$

$\underline{\mathbf{x_{1,2} = \pm 3;}}$ $\underline{\mathbf{x_{3,4} = \pm 2}}$ $\underline{\mathbf{L = \{-3, -2, +2, +3\}}}$

$x^6 + 19x^3 - 216 = 0 \rightarrow x^3 = u$

$u^2 + 19u - 216 = 0$

$$u_{1,2} = = \frac{-19}{2} \pm \sqrt{\left(\frac{19}{2}\right)^2 + 216}$$

$$u_{1,2} = \frac{-19}{2} \pm \sqrt{\frac{361}{4} + \frac{864}{4}}$$

$$u_{1,2} = \frac{-19}{2} \pm \sqrt{\frac{1225}{4}}; \quad u_{1,2} = \frac{-19}{2} \pm \frac{35}{2};$$

$\mathbf{u_1 = +8;}$ $\mathbf{u_2 = -27;}$ $x^3 = u;$

$x^3 = +8$ $x^3 = -27$

$\underline{\mathbf{x_1 = +2;}}$ $\underline{\mathbf{x_2 = -3;}}$ $\underline{\mathbf{L = \{-3, +2\}}}$

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