

Arithmetik – Algebraische Gleichungen höheren Grades

Lösungswege 2 von 2

Lösen Sie folgende Gleichungen über die Grundmenge $G = \mathbb{C}$!

*) Substitutionsmethode!

$(x^2 - 4x)^2 - 3x \cdot (4 - x) = 4$ $(x^2 - 4x)^2 + 3(x^2 - 4x) = 4$ <p style="text-align: right;">$\rightarrow \rightarrow \rightarrow$ <u>1. Schritt:</u></p>	<p>Man setzt für $(x^2 - 4x) \rightarrow u$ $(x^2 - 4x) = u$</p>
$u^2 + 3 \cdot u = 4$ $u^2 + 3u - 4 = 0$ $u_{1,2} = \frac{-3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 + 4}$ $u_{1,2} = \frac{-3}{2} \pm \sqrt{\frac{9}{4} + \frac{16}{4}}$ $u_{1,2} = \frac{-3}{2} \pm \frac{5}{2}; \quad \underline{u_1 = +1}; \quad \underline{u_2 = -4};$ <p style="text-align: right;">$\rightarrow \rightarrow \rightarrow$ <u>3. Schritt:</u></p>	<p>Lösung der quadratischen Gleichung!</p>
$(x^2 - 4x) = +1 \rightarrow x^2 - 4x - 1 = 0$ $x_{1,2} = \frac{4}{2} \pm \sqrt{\left(\frac{4}{2}\right)^2 + 1}; \quad \underline{x_{1,2} = +2 \pm \sqrt{5}};$ $\underline{x_1 = -0,24}; \quad \underline{x_2 = +4,76};$	$(x^2 - 4x) = -4 \rightarrow x^2 - 4x + 4 = 0$ $x_{3,4} = \frac{4}{2} \pm \sqrt{\left(\frac{4}{2}\right)^2 - 4}; \quad \underline{x_{3,4} = +2}$ <p style="text-align: right;"><u>$L = \{-0,24, +2, +4,76\}$</u></p>
$x^4 - 13x^2 + 36 = 0 \rightarrow x^2 = u$ $u^2 - 13u + 36 = 0$ $u_{1,2} = \frac{13}{2} \pm \sqrt{\left(\frac{13}{2}\right)^2 - 36}$ $u_{1,2} = \frac{13}{2} \pm \sqrt{\frac{169}{4} - \frac{144}{4}}$ $u_{1,2} = \frac{13}{2} \pm \sqrt{\frac{25}{4}}; \quad u_{1,2} = \frac{13}{2} \pm \frac{5}{2}$ $\underline{u_1 = +9}; \quad \underline{u_2 = +4}; \quad \underline{x^2 = u};$ $x^2 = +9 \quad x^2 = +4$ $\underline{x_{1,2} = \pm 3}; \quad \underline{x_{3,4} = \pm 2} \quad \underline{L = \{-3, -2, +2, +3\}}$	$x^6 + 19x^3 - 216 = 0 \rightarrow x^3 = u$ $u^2 + 19u - 216 = 0$ $u_{1,2} = \frac{-19}{2} \pm \sqrt{\left(\frac{19}{2}\right)^2 + 216}$ $u_{1,2} = \frac{-19}{2} \pm \sqrt{\frac{361}{4} + \frac{864}{4}}$ $u_{1,2} = \frac{-19}{2} \pm \sqrt{\frac{1225}{4}}; \quad u_{1,2} = \frac{-19}{2} \pm \frac{35}{2};$ $\underline{u_1 = +8}; \quad \underline{u_2 = -27}; \quad \underline{x^3 = u};$ $x^3 = +8 \quad x^3 = -27$ $\underline{x_1 = +2}; \quad \underline{x_2 = -3}; \quad \underline{L = \{-3, +2\}}$