Arithmetik – Algebraische Gleichungen höheren Grades

Lösungsblatt 3

Lösen Sie folgende Gleichungen über die Grundmenge die G = C durch Substitution!

$a^4 - 5a^2 + 4 = 0 \quad \rightarrow \quad a^2 = u$

$u^2 - 5u + 4 = 0$

$u_{1,2} = = \frac{5}{2} \pm \sqrt{\left(\frac{5}{2}\right)^2 - 4}$

$u_{1,2} = \frac{5}{2} \pm \sqrt{\frac{25}{4} - \frac{16}{4}}$

$u_{1,2} = \frac{5}{2} \pm \sqrt{\frac{9}{4}}; \quad u_{1,2} = \frac{5}{2} \pm \frac{3}{2}$

$\mathbf{u_1 = +4}; \quad \mathbf{u_2 = +1}; \quad a^2 = u;$

$a^2 = +4 \qquad a^2 = +1$

$\mathbf{a_{1,2} = \pm 2}; \quad \mathbf{a_{3,4} = \pm 1}$

L = {– 2, – 1, + 1, + 2}

$b^6 - 28b^3 + 27 = 0 \quad \rightarrow \quad b^3 = u$

$u^2 - 28u + 27 = 0$

$u_{1,2} = = \frac{28}{2} \pm \sqrt{\left(\frac{28}{2}\right)^2 - 27}$

$u_{1,2} = 14 \pm \sqrt{196 - 27}$

$u_{1,2} = 14 \pm \sqrt{169}; \quad u_{1,2} = 14 \pm 13;$

$\mathbf{u_1 = +27}; \quad \mathbf{u_2 = +1}; \quad b^3 = u;$

$b^3 = +27 \qquad b^3 = +1$

$\underline{\mathbf{b_1 = +3}}; \quad \underline{\mathbf{b_2 = +1}};$

L = {+ 1, + 3}

$2s^6 + 38s^3 = +432 \qquad \| -432; \,|\, : 2$

$s^6 + 19s^3 - 216 = 0 \quad \rightarrow \quad s^3 = z$

$z^2 + 19z - 216 = 0$

$z_{1,2} = = -\frac{19}{2} \pm \sqrt{\left(\frac{19}{2}\right)^2 + 216}$

$z_{1,2} = -\frac{19}{2} \pm \sqrt{\frac{361}{4} + \frac{864}{4}}$

$z_{1,2} = -\frac{19}{2} \pm \sqrt{\frac{1225}{4}}; \quad z_{1,2} = -\frac{19}{2} \pm \frac{35}{2}$

$\mathbf{z_1 = -\frac{19}{2} + \frac{35}{2}}; \quad \mathbf{z_2 = -\frac{19}{2} - \frac{35}{2}}; \quad z^3 = u;$

$s^3 = +8 \qquad s^3 = -27$

$\underline{\mathbf{s_1 = +2}}; \quad \underline{\mathbf{s_2 = -3}};$

L = {– 3, + 2}

$(m-5)^4 + (m^2 - 10m + 25) = 2$

$\rightarrow \quad (m^2 - 10m + 25) = (m-5)^2$

$(m-5)^4 + (m-5)^2 - 2 = 0 \quad \rightarrow \quad (m-5)^2 = u$

$u^2 + u - 2 = 0$

$u_{1,2} = = -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + 2}$

$u_{1,2} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{8}{4}}$

$u_{1,2} = -\frac{1}{2} \pm \sqrt{\frac{9}{4}}; \quad u_{1,2} = -\frac{1}{2} \pm \frac{3}{2}$

$\mathbf{u_1 = +1}; \quad \mathbf{u_2 = -2}; \quad (m-5)^2 = u;$

$(m-2)^2 = +1 \qquad \| \; (m-2)^2 = -2$

$m^2 - 10m + 24 = 0 \qquad \| \; m^2 - 10m + 27 = 0$

$m_{1,2} = \frac{10}{2} \pm \sqrt{\left(\frac{10}{2}\right)^2 - 24} \qquad \| \; m_{3,4} = \frac{10}{2} \pm \sqrt{\left(\frac{10}{2}\right)^2 - 27}$

$m_{1,2} = 5 \pm \sqrt{25 - 24} \qquad \| \; m_{3,4} = 5 \pm \sqrt{25 - 27}$

$m_{1,2} = 5 \pm 1; \qquad \| \; \mathbf{m_3 = 5 + \sqrt{2} \cdot i}$

$\mathbf{m_1 = +6}; \quad \mathbf{m_2 = +4}; \qquad \| \; \mathbf{m_4 = 5 - \sqrt{2} \cdot i}$

L = {+ 4, + 6, / + 5 + $\sqrt{2}$. i / + 5 – $\sqrt{2}$. i }