

Funktionen – Integralrechnen mit der Substitutionsmethode

Lösungsblatt 1

Berechnen Sie die unbestimmten Integrale mit Hilfe der Substitutionsmethode!

$$\begin{aligned}\int (4x + 5)^3 \cdot dx &= && \text{Substitution} \rightarrow (4x + 5) = z \rightarrow z' = 4 \\ &&& \rightarrow z' = \frac{dz}{dx} = 4 \rightarrow dx = \frac{dz}{4} \\ &= \int z^3 \cdot \frac{dz}{4} = \frac{1}{4} \cdot z^4 \cdot \frac{1}{4} + c = \frac{1}{16} \cdot z^4 + c = \frac{1}{16} \cdot (4x + 5)^4 + c\end{aligned}$$

$$\begin{aligned}\int \frac{2x}{\sqrt{x^2 - 4}} \cdot dx &= && \text{Substitution} \rightarrow (x^2 - 4) = z \rightarrow z' = 2x \\ &&& \rightarrow z' = \frac{dz}{dx} = 2x \rightarrow dx = \frac{dz}{2x} \\ &= \int \frac{2x}{\sqrt{z}} \cdot \frac{dz}{2x} = \int \frac{1}{\sqrt{z}} \cdot dz = \int z^{-\frac{1}{2}} \cdot dz \\ &&& = 2 \cdot z^{\frac{1}{2}} + c = 2 \cdot \sqrt{z} + c = 2 \cdot \sqrt{x^2 - 4} + c\end{aligned}$$

$$\begin{aligned}\int \sqrt{6x - 5} \cdot dx &= && \text{Substitution} \rightarrow (6x - 5) = z \rightarrow z' = 6 \\ &&& \rightarrow z' = \frac{dz}{dx} = 6 \rightarrow dx = \frac{dz}{6} \\ &= \int \sqrt{z} \cdot \frac{dz}{6} = \int z^{\frac{1}{2}} \cdot \frac{dz}{6} = \frac{2}{3} \cdot z^{\frac{3}{2}} \cdot \frac{1}{6} + c = \frac{2}{18} \cdot \sqrt{z^3} + c = \frac{1}{9} \cdot z \cdot \sqrt{z} + c\end{aligned}$$

$$\begin{aligned}\int (5x^2 - 2)^4 \cdot dx &= && \text{Substitution} \rightarrow (5x^2 - 2) = z \rightarrow z' = 10 \\ &&& \rightarrow z' = \frac{dz}{dx} = 10 \rightarrow dx = \frac{dz}{10} \\ &= \int z^4 \cdot \frac{dz}{10} = \frac{1}{5} \cdot z^5 \cdot \frac{1}{10} + c = \frac{1}{50} \cdot z^5 + c = \frac{1}{50} \cdot (5x^2 - 2)^5 + c\end{aligned}$$