Funktionen – Anwendung der Integralrechnen – Volumen von Rotationskörpern

Lösungsblatt 2

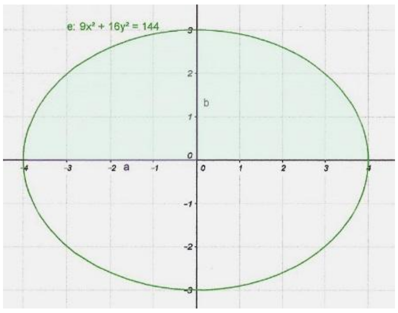
Berechnen Sie das Volumen des Rotationskörpers, der bei Rotation der Ellipse
ϵ: $9x^2 + 16y^2 = 144$ a *) um die x-Achse b *) um die y-Achse entsteht!

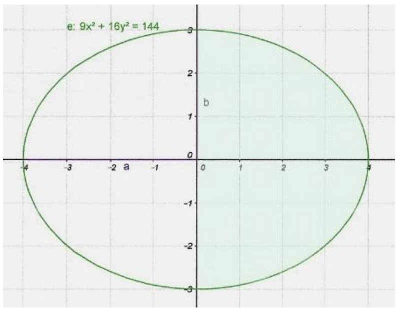
ϵ: $b^2x^2 + a^2y^2 = a^2 \cdot b^2$

ϵ: $\underline{9x^2 + 16y^2 = 144}$ $\rightarrow$ $a^2 = 16$; $a = 4$;

$\rightarrow$ $b^2 = 9$; $b = 3$; → siehe Skizze!

$x^2 = 16 - \frac{16}{9}y^2$; $y^2 = 9 - \frac{9}{16}x^2$;





<u>Rotation um die x–Achse</u>:

$$V_x = \pi \cdot \int_{-a}^{+a} y^2 \cdot dx = \pi \cdot \int_{-4}^{+4} (9 - \frac{9}{16}x^2) \cdot dx =$$
$$= \pi \cdot (9x - \frac{9}{16 \cdot 3}x^3)\Big|_{-4}^{+4} =$$
$$= \pi \cdot [(36 - \frac{3 \cdot 64}{16}) - (-36 - \frac{3 \cdot (-16)}{16})] =$$
$$= \pi \cdot [(36 - 12) - (-36 + 12)] = (24 + 24) \cdot \pi = \underline{48 \cdot \pi}$$

<u>Rotation um die y–Achse:</u>

$$V_y = \pi \cdot \int_{-b}^{+b} x^2 \cdot dy = \pi \cdot \int_{-3}^{+3} (16 - \frac{16}{9}y^2) \cdot dy =$$
$$= \pi \cdot (16y - \frac{16}{9 \cdot 3}y^3)\Big|_{-3}^{+3} =$$
$$= \pi \cdot [(48 - \frac{16 \cdot 27}{27}) - (-48 - \frac{16 \cdot (-27)}{27})] =$$
$$= \pi \cdot [(48 - 16) - (-48 + 16)] = (32 + 32) \cdot \pi = \underline{64 \cdot \pi}$$

$\underline{V_x = 48 \cdot \pi}$ VE $\underline{V_y = 64 \cdot \pi}$ VE $V_x : V_y = 48 : 64 = 3 : 4$

Ebenso: ϵ: $4x^2 + 25y^2 = 100$; $V_x = ?$; $V_y = ?$;

ϵ: $b^2x^2 + a^2y^2 = a^2 \cdot b^2$ $\rightarrow$ $a^2 = 25$; $a = 5$; $\rightarrow$ $b^2 = 4$; $b = 2$;

ϵ: $\underline{4x^2 + 25y^2 = 100}$ $\rightarrow$ $x^2 = 25 - \frac{25}{4}y^2$; $\rightarrow$ $y^2 = 4 - \frac{4}{25}x^2$;

<u>Rotation um die x–Achse</u>:

$$V_x = \pi \cdot \int_{-a}^{+a} y^2 \cdot dx = \pi \cdot \int_{-5}^{+5} (4 - \frac{4}{25}x^2) \cdot dx = \pi \cdot (4x - \frac{4}{25 \cdot 3}x^3)\Big|_{-5}^{+5} =$$
$$= \pi.[(20 - \frac{4 \cdot 125}{75}) - (-20 - \frac{4 \cdot (-125)}{75})] = \pi.[(20 - \frac{20}{3}) - (-20 + \frac{20}{3})] = (\frac{40}{3} + \frac{40}{3}).\pi = \mathbf{(\frac{80}{3}).\pi}$$

<u>Rotation um die y–Achse:</u>

$$V_y = \pi \cdot \int_{-b}^{+b} x^2 \cdot dy = \pi \cdot \int_{-2}^{+2} (25 - \frac{25}{4}y^2) \cdot dy = \pi \cdot (25y - \frac{25}{4 \cdot 3}y^3)\Big|_{-2}^{+2} =$$
$$= \pi \cdot [(50 - \frac{25 \cdot 8}{12}) - (-50 - \frac{25 \cdot (-8)}{12})] = \pi \cdot [(50 - \frac{50}{3}) - (-50 + \frac{50}{3})] = (\frac{100}{3} + \frac{100}{3}).\pi = \mathbf{\frac{200}{3}.\pi}$$

$V_x = \mathbf{(\frac{80}{3}).\pi}$ VE $V_y = \mathbf{\frac{200}{3}.\pi}$ VE $V_x : V_y = 80 : 200 = 2 : 3$