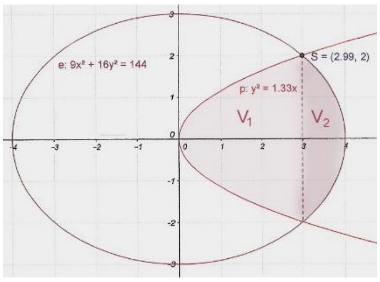
Funktionen – Anwendung der Integralrechnen – Volumen von Rotationskörpern

Lösungsblatt 4

Der Ellipse / (die Hyperbel) / schneidet von der Parabel ein Flächenstück ab. Dieses Flächenstück rotiert um die x-Achse. Berechnen Sie das Volumen des Rotationskörpers!

V_1 = Rotation der Parabel im Intervall (0; x_s)
V_2 = Rotation des Ellipse im Intervall (x_s; a)



ϵ: $9x^2 + 16y^2 = 144;$ $\rightarrow$ $a^2 = 16; a = 4;$ $\rightarrow$ $b^2 = 9; b = 3;$

p: $y^2 = \frac{4}{3}x;$ $\rightarrow$ $y^2 = 9 - \frac{9}{16}x^2;$

$\rightarrow$ $\epsilon \cap p$: $\rightarrow$ $9x^2 + 16 \cdot \frac{4}{3}x - 144 = 0$ $\rightarrow$ $|\cdot 3$

$27x^2 + 64 - 432;$

$$x_{1,2} = \frac{-64 \pm \sqrt{(-64)^2-(4.27.-144)}}{2.27} = \frac{-64 \pm 225{,}282}{54};$$

$\mathbf{x_1 = +2{,}99 \approx 3;}$ $[x_2 = -]$

$V = V_1 + V_2$

$V_1 = \pi \cdot \int_a^b y^2 \cdot dx = \pi \cdot \int_0^{+3} \frac{4}{3}x \cdot dx = \pi \cdot (\frac{4}{6}x^2)\Big|_0^{+3} =$

$= \pi \cdot (6 - 0) = \underline{\mathbf{6 \cdot \pi}}$

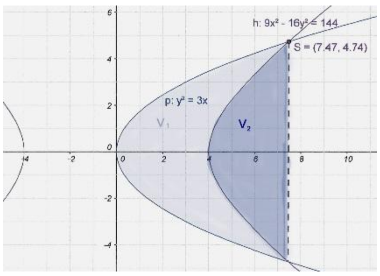
$V_2 = \pi \cdot \int_a^b y^2 \cdot dx = \pi \cdot \int_{+3}^{+4} (9 - \frac{9}{16}x^2) \cdot dx =$

$= \pi \cdot (9x - \frac{9}{48}x^3)\Big|_{+3}^{+4} = \pi \cdot [(36 - \frac{9}{48} \cdot 64) - (27 - \frac{9}{48} \cdot 27)] =$

$= (24 - 21{,}95) \cdot \pi = \underline{\mathbf{2{,}05 \cdot \pi}}$

$V = (6 + 2{,}05) \cdot \pi =$ $\underline{\mathbf{V = 25{,}28\ VE}}$

V_1 = Rotation der Parabel im Intervall (0; x_s)
V_2 = Rotation des Ellipse im Intervall (a; x_s)



h: $9x^2 - 16y^2 = 144;$ $\rightarrow$ $a^2 = 16; a = 4;$ $\rightarrow$ $b^2 = 9; b = 3;$

p: $y^2 = 3x;$ $\rightarrow$ $y^2 = \frac{9}{16}x^2 - 9;$

$\rightarrow$ $h \cap p$: $\rightarrow$ $9x^2 - 16 \cdot 3x - 144 = 0$ $\rightarrow$ $|:9$

$x^2 - \frac{16}{3}x - 16 = 0$

$$x_{1,2} = \frac{8}{3} \pm \sqrt{\left(\frac{8}{3}\right)^2 + 16} = \frac{8}{3} \pm \sqrt{\frac{64}{9} + \frac{144}{9}} = \frac{8}{3} \pm \sqrt{\frac{208}{9}}$$

$\mathbf{x_1} = +2{,}6 + 4{,}8 = \mathbf{7{,}4;}$ $[x_2 = -]$

$V = V_1 - V_2$

$V_1 = \pi \cdot \int_a^b y^2 \cdot dx = \pi \cdot \int_0^{+7,4} (3x) \cdot dx = \pi \cdot (\frac{3}{2}x^2)\Big|_0^{+7,4} =$

$= \pi \cdot (\frac{164{,}28}{2} - 0) = \underline{\mathbf{82{,}14 \cdot \pi}}$

$V_2 = \pi \cdot \int_a^b y^2 \cdot dx = \pi \cdot \int_{+3}^{+4} (\frac{9}{16}x^2 - 9) \cdot dx =$

$= \pi \cdot (\frac{9}{48}x^3 - 9x)\Big|_{+4}^{+7,4} = \pi \cdot [(-75{,}9 - 66{,}6) - (12 - 36)] =$

$= (9{,}4 + 24) \cdot \pi = \underline{\mathbf{33{,}4 \cdot \pi}}$

$V = (82{,}14 - 33{,}4) \cdot \pi$ $\underline{\mathbf{V = 153{,}43\ VE}}$